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THE TRANSPORT PROPERTIES OF GASES IN BURNETT'S APPROXIMATION†

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The relations necessary to calculate Burnett's corrections to the distribution functions of the molecules of multicomponent mixtures of polyatomic gases are derived. Effective expressions for the transport properties and "working" formulae for the transport coefficients of a binary mixture of monatomic gases and a polyatomic gas in Burnett's approximation ignoring external forces are obtained. The transient equations of thermal stress convection of a polyatomic gas are considered and estimates of the effect of the rotational degrees of freedom of the molecules on the coefficients of these equations are given.

Supplementing a previous paper [1], we present below results which are of independent interest (for similar data for monatomic gas see [2, 3]), and we also consider more completely the most important case of a binary mixture of monatomic gases and a polyatomic gas. In the latter case, the general equations of thermal stress convection and concentration stress convection [1] are converted to a much simpler form. The impulse equation of thermal stress convection finally obtained contains terms resulting from Burnett temperature stresses. The results of estimates of the effect of the rotational degrees of freedom of the molecules on the coefficients of these terms are necessary, in particular, for analysing experimental data [4]. Linear problems of the sound propagation and the structure of a weak shock wave in polyatomic gas were solved previously in [5] using Burnett's equations. Unless otherwise stated, we use the notation employed previously in [1].

1. THE EQUATIONS FOR THE BURNETT CORRECTIONS TO THE DISTRIBUTION FUNCTIONS

The system of equations for the Burnett corrections to the distribution functions $f_\Omega^{(2)} = f_\Omega^{(0)}\varphi_\Omega^{(2)}$ were written in [1] in the form

$$M_\Omega = n^2 R_\Omega(\varphi^{(2)}), \quad M_\Omega \equiv \frac{\partial_1 f_\Omega^{(0)}}{\partial t} + H_\Omega - L_\Omega(f^{(1)}f^{(1)}) \tag{1.1}$$

The first two terms of the inhomogeneous part M_Ω of Eq. (1.1) are due to the convection terms of the kinetic equation, where H_Ω is a group of terms expressed in terms of the natural velocities of the molecules $\mathbf{c}_i$ and containing derivative of $f_\Omega^{(1)}$. As described previously in [2, 3] these two terms are represented by three groups of terms.

The first group depends only on the scalar c_i

$$f_\Omega^{(0)}\left\{-\frac{1}{n_i}\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{J}_i^{(1)} + \frac{2}{3pc_v^*}\left(\frac{3}{2} - w_i^2 - \Delta\varepsilon_\Omega\right)\left[\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{q}^{(1)} + \boldsymbol{\tau}^{(1)} : \frac{\partial}{\partial \mathbf{r}}\mathbf{u} - \right.\right.$$
$$\left.\left. -\sum_{j=1}^{S}\left(\mathbf{J}_j\cdot\mathbf{F}_j + E_j^*\frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{J}_j\right)\right]\right\} - \frac{2T}{3c_v^*}\frac{\partial\Gamma'_\Omega}{\partial T}(\nabla\mathbf{u})^2 + \Gamma'_\Omega\frac{D_0\nabla\mathbf{u}}{Dt} +$$
$$+\mathbf{Z}_i\left(A'_\Omega\frac{\partial T}{\partial \mathbf{r}} + \sum_{j=1}^{S} D'^j_\Omega\mathbf{d}_j\right), \quad \nabla\mathbf{u} \equiv \frac{\partial}{\partial \mathbf{r}}\cdot\mathbf{u} \tag{1.2}$$

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The second group consists of terms of an odd power of the vector $\mathbf{c}_i$

$$-\frac{m_i}{kT\rho}f_\Omega^{(0)}\mathbf{c}_i\frac{\partial}{\partial\mathbf{r}}:\boldsymbol{\tau}^{(1)}-\frac{2}{3}\left(\mathbf{c}_i\cdot\frac{\partial T}{\partial\mathbf{r}}\right)\left(\frac{T}{c_v^*}\frac{\partial A'_\Omega}{\partial T}+c_i^2\frac{\partial A'_\Omega}{\partial c_i^2}\right)\nabla\mathbf{u}+$$

$$+A'_\Omega\mathbf{c}_i\cdot\left[\frac{D_0}{Dt}\left(\frac{\partial T}{\partial\mathbf{r}}\right)-\left(\frac{\partial}{\partial\mathbf{r}}\mathbf{c}_i\right)\cdot\frac{\partial T}{\partial\mathbf{r}}\right]-\frac{2}{3}\mathbf{c}_i\cdot\sum_{j=1}^{s}\mathbf{d}_j\left(\frac{T}{c_v^*}\frac{\partial D'^j_\Omega}{\partial T}+c_i^2\frac{\partial D'^j_\Omega}{\partial c_i^2}\right)\nabla\mathbf{u}+$$

$$+\sum_{j=1}^{s}D'^j_\Omega\mathbf{c}_i\cdot\left[\frac{D_0\mathbf{d}_j}{Dt}-\left(\frac{\partial}{\partial\mathbf{r}}\mathbf{u}\right)\cdot\mathbf{d}_j\right]+$$

$$+2\mathbf{Z}_i\cdot\left\{\frac{\partial B'_\Omega}{\partial c_i^2}\mathbf{c}_i(\mathbf{c}_i\mathbf{c}_i:\mathbf{e})+B'_\Omega\mathbf{e}\cdot\mathbf{c}_i+\mathbf{c}_i\frac{\partial\Gamma'_\Omega}{\partial c_i^2}\nabla\mathbf{u}\right\}+$$

$$+(\mathbf{c}_i\mathbf{c}_i:\mathbf{e})\mathbf{c}_i\cdot\left\{\left(\frac{\partial B'_\Omega}{\partial T}-2\frac{\partial A'_\Omega}{\partial c_i^2}\right)\frac{\partial T}{\partial\mathbf{r}}+\sum_{j=1}^{s}\left(\frac{\partial B'_\Omega}{\partial x_j}\frac{\partial x_j}{\partial\mathbf{r}}-2\frac{\partial D'^j_\Omega}{\partial c_i^2}\mathbf{d}_j\right)\right\}+$$

$$+B'_\Omega\mathbf{c}_i\cdot\frac{\partial}{\partial\mathbf{r}}(\mathbf{c}_i\mathbf{c}_i:\mathbf{e})+\mathbf{c}_i\cdot\left(\frac{\partial\Gamma'_\Omega}{\partial T}\frac{\partial T}{\partial\mathbf{r}}+\sum_{j=1}^{s}\frac{\partial\Gamma'_\Omega}{\partial x_j}\frac{\partial x_j}{\partial\mathbf{r}}\right)\nabla\mathbf{u}+\Gamma'_\Omega\mathbf{c}_i\cdot\frac{\partial\nabla\mathbf{u}}{\partial\mathbf{r}} \qquad (1.3)$$

Finally, the third group contains terms of an even power of the vector $\mathbf{c}_i$

$$-\frac{2}{3}(\mathbf{c}_i\mathbf{c}_i:\mathbf{e})\left(\frac{T}{c_v^*}\frac{\partial B'_\Omega}{\partial T}+c_i^2\frac{\partial B'_\Omega}{\partial c_i^2}\right)\nabla\mathbf{u}+B'_\Omega\mathbf{c}_i\mathbf{c}_i:\left[\frac{D_0\mathbf{e}}{Dt}-2\left(\frac{\partial}{\partial\mathbf{r}}\mathbf{u}\right)\cdot\mathbf{e}\right]+$$

$$+\mathbf{c}_i\mathbf{c}_i:\left[A'_\Omega\frac{\partial}{\partial\mathbf{r}}\frac{\partial T}{\partial\mathbf{r}}+\sum_{j=1}^{s}D'^j_\Omega\frac{\partial}{\partial\mathbf{r}}\mathbf{d}_j+\frac{\partial T}{\partial\mathbf{r}}\left(\frac{\partial A'_\Omega}{\partial T}\frac{\partial T}{\partial\mathbf{r}}+\sum_{j=1}^{s}\frac{\partial A'_\Omega}{\partial x_j}\frac{\partial x_j}{\partial\mathbf{r}}\right)+\right.$$

$$+\sum_{j=1}^{s}\mathbf{d}_j\left(\frac{\partial D'^j_\Omega}{\partial T}\frac{\partial T}{\partial\mathbf{r}}+\sum_{k=1}^{s}\frac{\partial D'^j_\Omega}{\partial x_k}\frac{\partial x_k}{\partial\mathbf{r}}\right)+2\mathbf{Z}_i\left(\frac{\partial A'_\Omega}{\partial c_i^2}\frac{\partial T}{\partial\mathbf{r}}+\sum_{j=1}^{s}\frac{\partial D'^j_\Omega}{\partial c_i^2}\mathbf{d}_j\right)-$$

$$\left.-2\left(\frac{\partial}{\partial\mathbf{r}}\mathbf{u}\right)\frac{\partial\Gamma'_\Omega}{\partial c_i^2}\nabla\mathbf{u}\right]-2\frac{\partial B'_\Omega}{\partial c_i^2}(\mathbf{c}_i\mathbf{c}_i:\mathbf{e})(\mathbf{c}_i\mathbf{c}_i:\mathbf{e}) \qquad (1.4)$$

In expressions (1.2)–(1.4) we have used the notation [2] for the vector and tensor operations, which simplifies a comparison with the case of a monatomic gas and the use of integral theorems [2, 3], but instead of $\mathbf{p}^{(1)}$, $\mathbf{c}_0$, $\mathring{\mathbf{e}}$, ∇ the corresponding symbols [1] $\boldsymbol{\tau}^{(1)}$, $\mathbf{u}$, $\mathbf{e}$, $\nabla\mathbf{u}$ are retained. In the first (Navier–Stokes) approximation for the transport properties we have, similar to the traditional approach [3]

$$\boldsymbol{\tau}^{(1)}=\Pi^{(1)}\boldsymbol{\delta}+\boldsymbol{\pi}^{(1)},\quad \Pi^{(1)}=-\zeta\nabla\mathbf{u},\quad \boldsymbol{\pi}^{(1)}=-2\eta\mathbf{e}$$

$$\mathbf{V}_i^{(1)}=-\sum_{j=1}^{s}D_{ij}\mathbf{d}_j-D_{Ti}\frac{\partial\ln T}{\partial\mathbf{r}},\quad (\zeta,\eta,\lambda')=\sum_{i=1}^{s}(\zeta_i,\eta_i,\lambda'_i) \qquad (1.5)$$

$$\mathbf{h}^{(1)}=-\lambda'\frac{\partial T}{\partial\mathbf{r}}-p\sum_i D_{Ti}\mathbf{d}_i,\quad \mathbf{q}^{(1)}=\mathbf{h}^{(1)}+kT\sum_{i=1}^{s}\left(\frac{5}{2}+\langle\varepsilon_\Omega\rangle_c\right)n_i\mathbf{V}_i^{(1)}$$

Here $\boldsymbol{\delta}$ is the unit tensor. Relations (1.5) define the transport coefficients of the Navier–Stokes equations used previously [1]. The third term in the expression for M_Ω contains the collision operator L_Ω.

A knowledge of the quantity $f_\Omega^{(2)}$ is also necessary when considering the super-Burnett approximation and in the theory of a kinetic (Knudsen) layer. To determine the Burnett contributions to the transport properties we do not need to calculate $f_\Omega^{(2)}$; it is sufficient to calculate certain moments of the operation M_Ω, which is expressed in terms of the Maxwellian $f_\Omega^{(0)}$ and

$$f_\Omega^{(1)} = -\frac{1}{n} f_\Omega^{(0)}\left(A_\Omega c_{i\alpha} \frac{\partial \ln T}{\partial r_\alpha} + B_\Omega c_{i\alpha} c_{i\beta} e_{\alpha\beta} + \sum_{j=1}^{S} D_\Omega^j c_{i\alpha} d_{j\alpha} + \Gamma_\Omega \nabla \mathbf{u} \right) \equiv$$

$$\equiv A'_\Omega c_{i\alpha} \frac{\partial T}{\partial r_\alpha} + B'_\Omega c_{i\alpha} c_{i\beta} e_{\alpha\beta} + \sum_{j=1}^{S} D'^j_\Omega c_{i\alpha} d_{j\alpha} + \Gamma'_\Omega \nabla \mathbf{u} \tag{1.6}$$

In expression (1.6) and below, unlike (1.2)–(1.4), we use the component-wise form [1] of the vector and tensor quantities and the usual rule for summation over repeated subscripts. The components of the radius vector $\mathbf{r}$ are introduced by the subscripts α, β and γ, and the operators

$$\langle N_{\alpha\beta} \rangle = \frac{1}{2}(N_{\alpha\beta} + N_{\beta\alpha}) - \frac{1}{3}\delta_{\alpha\beta} N_{\gamma\gamma}, \quad e_{\alpha\beta} = \left\langle \frac{\partial u_\alpha}{\partial r_\beta} \right\rangle, \quad \nabla \mathbf{u} = \frac{\partial u_\gamma}{\partial r_\gamma} \tag{1.7}$$

2. THE FIRST APPROXIMATION FOR A BINARY MIXTURE OF MONATOMIC GASES

We will give the information required to transfer from the general case [1] to the case being considered, and the results of the first approximation which will be required later. The latter is dictated by the need to write expressions for the partial transport coefficients in a form convenient for use (the overall transport coefficients of the mixture of gases occur in the Navier–Stokes equations) in the approximation assumed earlier [1] in terms of Sonin polynomials, relating these expressions to the data in the most quoted book [3]. In the case of a mixture of monatomic gases we must put Y_Ω, c_v^* equal to unity in the formulae in [1], put ε_Ω, $\Delta\varepsilon_\Omega$, c_{vi}, Γ_Ω, Π, ζ_i, λ_{vi} equal zero, and replace the subscript Ω by i. We then obtain

$$\tau_{\alpha\beta} = \pi_{\alpha\beta}, \quad E_i^* = \frac{3}{2}kT, \quad U = \frac{1}{2}m_i c_i^2, \quad \lambda'_{ti} = \lambda'_i$$

In the case of a binary mixture $x_1 + x_2 = 1$, $d_{1\alpha} = -d_{2\alpha}$, therefore, for example

$$\sum_{j=1}^{2} D_i^j d_{j\alpha} = \mathcal{D}_i d_{1\alpha}, \quad \sum_{j,k=1}^{2} \frac{\partial D_i^j}{\partial x_k} d_{j\alpha} \frac{\partial x_k}{\partial r_\beta} = 2\frac{\partial \mathcal{D}_i}{\partial x_1} d_{1\alpha} \frac{\partial x_1}{\partial r_\beta} \tag{2.1}$$

$$\mathcal{D}_i = D_i^1 - D_i^2, \quad i = 1,2 \tag{2.2}$$

Then, instead of (1.6), we obtain

$$f_i^{(1)} = -\frac{1}{n} f_i^{(0)}\left(A_i c_{i\alpha} \frac{\partial \ln T}{\partial r_\alpha} + B_i c_{i\alpha} c_{i\beta} e_{\alpha\beta} + \mathcal{D}_i c_{i\alpha} d_{1\alpha} \right) \equiv$$

$$\equiv A'_i c_{i\alpha} \frac{\partial T}{\partial r_\alpha} + B'_i c_{i\alpha} c_{i\beta} e_{\alpha\beta} + \mathcal{D}'_i c_{i\alpha} d_{1\alpha} \tag{2.3}$$

The Chapman–Enskog method gives series for the stresses, the diffusion rates and the thermal flux

$$\pi_{\alpha\beta} = \pi_{\alpha\beta}^{(1)} + \pi_{\alpha\beta}^{(2)} + \ldots, \quad V_{1\alpha} = V_{1\alpha}^{(1)} + V_{1\alpha}^{(2)} + \ldots, \quad V_{2\alpha} = -\frac{\rho_1}{\rho_2} V_{1\alpha}$$

$$q_\alpha = q_\alpha^{(1)} + q_\alpha^{(2)} + \ldots, \quad q_\alpha = h_\alpha + \frac{5}{2} p_1 \left(1 - \frac{m_1}{m_2}\right) V_{1\alpha} \tag{2.4}$$

The first terms correspond to the Navier–Stokes approximation and the second terms correspond to Burnett's approximation. We have used the following notation: m_i and n_i are the mass of the molecule and the number density of the ith component of the mixture, k is Boltzmann's constant, T is the temperature and $\mathbf{u}$ is the mean-mass velocity. Moreover

$$\rho_i = m_i n_i, \quad x_i = \frac{n_i}{n}, \quad p_i = n_i kT, \quad (n, \rho, p) = \sum_{i=1}^{2} (n_i, \rho_i, p_i) \tag{2.5}$$

$$\pi^{(1)}_{\alpha\beta} = -2\eta e_{\alpha\beta} \quad (\eta, \lambda') = \sum_{i=1}^{2} (\eta_i, \lambda'_i) \tag{2.6}$$

$$V^{(1)}_{1\alpha} = -\frac{m_2 n}{x_1 \rho}\mathcal{D}_{12}\left(d_{1\alpha} + k_T \frac{\partial \ln T}{\partial r_\alpha}\right), \quad k_T = \frac{\mathcal{D}_T}{\mathcal{D}_{12}} \tag{2.7}$$

$$h^{(1)}_\alpha = -\lambda\frac{\partial T}{\partial r_\alpha} + p\frac{\rho}{\rho_2}k_T V^{(1)}_{1\alpha}, \quad \lambda = \lambda' - \frac{nk}{x_1 x_2}\mathcal{D}_{12} k_T^2 \tag{2.8}$$

By definition [1] the partial coefficient of viscosity and thermal conductivity are given by the formulae

$$\eta_i = \frac{1}{2}kTx_i b_{i,0}, \quad \lambda'_i = \frac{5}{4}kx_i a_{i,1}$$

where $b_{i,0}$ and $a_{i,1}$ are the coefficients of the expansions in Sonin polynomials [3]. However, it was previously assumed [3] that η_i is the coefficient of viscosity of the gas of sort i. To eliminate the confusion we will denote the first approximations of the coefficient of viscosity and thermal conductivity of a monatomic gas of sort i in terms of Sonin polynomials as follows:

$$[\mathring{\eta}_i]_1 \equiv \mathring{\eta}_i = \frac{5}{16}\frac{(\pi m_i kT)^{1/2}}{\pi\sigma_i^2 \Omega_i^{(2,2)*}}, \quad [\mathring{\lambda}_i]_1 \equiv \mathring{\lambda}_i = \frac{15k}{4m_i}\mathring{\eta}_i \tag{2.9}$$

Here and below $\Omega_i^{(l,r)*}$ and $\Omega_{ij}^{(l,r)*}$ are the reduced Ω-integrals for a gas of sort i and mixtures of gases respectively, σ_i is the diameter of a molecule, and the brackets $[\ ]_n$ indicate the number (n) of the approximation in terms of Sonin polynomials [3]. To economize on space these brackets will be omitted as far as possible.

On changing from relations (1.5) to (2.7) and (2.8) we took into account the following formulae, which express the diffusion coefficient D_{ij} and the thermal diffusion coefficient D_{Ti} of a multicomponent mixture in terms of the diffusion coefficient $\mathcal{D}_{12}$ and the thermal diffusion coefficient $\mathcal{D}_T$ of a binary mixture

$$D_{12} = D_{21} = -\left(\frac{n}{\rho}\right)^2 m_1 m_2 \mathcal{D}_{12}, \quad D_{ii} = \frac{\rho_j}{\rho_i}\left(\frac{n}{\rho}\right)^2 m_1 m_2 \mathcal{D}_{12}$$

$$kn_i D_{Ti} = \omega_i \mathcal{D}_T, \quad \omega_i = (-1)^{i+1}\frac{n^2 k}{\rho m_i} m_1 m_2 \tag{2.10}$$

In formulae (2.10) and henceforth in (2.11)–(2.14) the subscripts i and j take the values 1 and 2, where $j \neq i$.

We will now change to approximate expressions for the transport coefficients. For B_i we will confine ourselves [1] to the first approximation in Sonin polynomials

$$B_i \approx \frac{m_i \eta_i}{x_i (kT)^2}, \quad \eta_i \equiv [\eta_i]_1 \tag{2.11}$$

where, taking expressions (2.9) into account

$$\eta_i = \mathring{\eta}_i x_i \Delta_\eta^{-1}(l_{jj} - l_{ij}), \quad \Delta_\eta = l_{22}l_{11} - l_{12}l_{21} \tag{2.12}$$

For the quantities l, using formulae (7.3.80) from [3] we have

$$l_{ii} = x_i + 2x_j \eta^*_{ij} M_1 M_2\left(\frac{5}{3A^*} + \frac{M_j}{M_i}\right) \tag{2.13}$$

$$l_{ij} = -2x_j \eta^*_{ji} M_1 M_2\left(\frac{5}{3A^*} - 1\right); \quad M_i = \frac{m_i}{m_1 + m_2}$$

We will denote by A^*, B^* and C^* the ratios of the reduced Ω-integrals [3] A^*_{12}, B^*_{12} and C^*_{12} respectively, equal to unity in the case of molecules (elastic spheres). For Maxwell molecules $A^* \approx 1.29$, $B^* \approx 1.25$ and $\sigma \approx 0$. The quantities

$$\eta_{ij}^{*}=\frac{\overset{\circ}{\eta}_{i}}{[\eta_{ij}]_{1}},\quad \lambda_{ij}^{*}=\frac{\overset{\circ}{\lambda}_{i}}{[\lambda_{ij}]_{1}} \tag{2.14}$$

$$[\eta_{ij}]_{1}=\frac{5nm_{i}M_{j}}{3A^{*}}[\mathcal{D}_{ij}]_{1},\quad [\lambda_{ij}]_{1}=\frac{15k}{8m_{i}M_{j}}[\eta_{ij}]_{1} \tag{2.15}$$

where $[\mathcal{D}_{ij}]_1$ is the diffusion coefficient of a binary mixture in the first approximation in terms of Sonin polynomials (formulae (7.3.38)[3]). The reduced integrals and the quantities A_{12}^*, B_{12}^*, C_{12}^*, $\lfloor\eta_{ij}\rfloor_1$, $\lfloor\lambda_{ij}\rfloor_1$, $\mathcal{D}_{ij}$ are independent of a permutation of the subscripts.

Approximate values of the Burnett transport coefficients were obtain earlier [1] in the second approximation in terms of Sonin polynomials for A_Ω, D_Ω^i. In this approximation, taking relations (2.3) and (2.10) into account and henceforth omitting the bracket $[\ ]_2$, we have

$$A_{i}\approx\frac{2m_{i}}{5k^{2}Tx_{i}}\left[\frac{5}{2}\omega_{i}\mathcal{D}_{T}-\lambda_{i}'S_{3/2}^{(1)}(w_{i}^{2})\right] \tag{2.16}$$

$$\mathcal{D}_{i}\approx\frac{m_{i}}{kT}\left[n(D_{i1}-D_{i2})+\delta\gamma_{i}S_{3/2}^{(1)}(w_{i}^{2})\right]$$

$$D_{i1}-D_{i2}\approx\frac{\omega_{i}}{kn_{i}}\mathcal{D}_{12},\quad \delta\gamma_{i}=\gamma_{i}^{1}-\gamma_{i}^{2} \tag{2.17}$$

For the coefficients occurring in (2.16) and (2.17) we obtain, using well-known results [3]

$$\mathcal{D}_{12}=\frac{[\mathcal{D}_{12}]_{1}}{1-\Delta},\quad \Delta=\frac{\sigma^{2}P_{3}}{10Q_{3}},\quad \sigma=6C^{*}-5 \tag{2.18}$$

$$\mathcal{D}_{T}=k_{T}\mathcal{D}_{12},\quad k_{T}=x_{1}x_{2}\sigma k_{T}^{*},\quad k_{T}^{*}=\frac{1}{Q_{3}}(S_{1}x_{1}-S_{2}x_{2}) \tag{2.19}$$

$$\lambda=[\lambda_{12}]_{1}\frac{P_{3}}{Q_{3}},\quad \lambda'=\lambda+\frac{nk}{x_{1}x_{2}}k_{T}^{2}\mathcal{D}_{12} \tag{2.20}$$

In relations (2.18)–(2.20) and (2.22) the functions P_3, Q_3 and R_3 are trinomials of the form $R_3 = R_1x_1^2 + R_2x_2^2 + R_{12}x_1x_2$. The coefficients

$$R_{1}=Q_{1}\lambda_{1}^{*},\quad R_{2}=Q_{2}\lambda_{2}^{*},\quad R_{12}=\frac{4A^{*}}{5M_{1}M_{2}}\left(\frac{1}{\lambda_{1}^{*}}+\frac{1}{\lambda_{2}^{*}}\right)+11-\frac{12}{5}B^{*}-\frac{16}{5}A^{*} \tag{2.21}$$

The coefficients $S_1, S_2, P_1, P_2, P_{12}, Q_1, Q_2$, and Q_{12}, are given by formulae (7.3.70), (7.3.43) and (7.3.44) from [3] respectively.

For λ_i' and $\delta\gamma_i$ it is convenient to use the following formulae (see, for example, [6]), expressed in terms of (2.9), (2.14), (2.15) and (2.18)–(2.21).

$$\lambda_{i}'=\frac{\rho_{i}}{\rho}\lambda'+(-1)^{i}5\frac{n^{2}}{\rho}(m_{1}+m_{2})kx_{1}x_{2}k_{T}^{*}\mathcal{D}_{12} \tag{2.22}$$

$$\delta\gamma_{i}=-\frac{n^{2}}{5\rho}m_{i}\sigma\mathcal{D}_{12}\left[2k_{T}^{*}+(-1)^{i}\frac{P_{3}M_{i}}{x_{i}M_{1}M_{2}Q_{3}}\right]$$

(errors which occur in [6] have been corrected).

In the first approximation (accurate for Maxwell molecules) we have $k_T = 0$, $\delta\gamma_i = 0$. In the case that is linear with respect to k_T we have $\lambda' = \lambda$.

External forces are ignored below. Then

$$d_{1\alpha}=\frac{\partial x_{1}}{\partial r_{\alpha}}+x_{1}x_{2}(m_{2}-m_{1})\frac{n}{\rho}\frac{\partial\ln p}{\partial r_{\alpha}} \tag{2.23}$$

$$\frac{D_{0}d_{1\alpha}}{Dt}=-\frac{\partial u_{\beta}}{\partial r_{\alpha}}d_{1\beta}-\frac{5}{3}x_{1}x_{2}(m_{2}-m_{1})\frac{n}{\rho}\frac{\partial\nabla\mathbf{u}}{\partial r_{\alpha}}$$

3. THE CONTRIBUTIONS OF BURNETT'S APPROXIMATION TO THE TRANSPORT PROPERTIES OF A BINARY MIXTURE OF MONATOMIC GASES

The formulae derived earlier [1, Section 4], were obtained from the general formulae with the thermal diffusion and barodiffusion effects ignored ($\sigma = 0$, $\mathbf{d}_1 = \nabla x_1$). We will derive both accurate and more complete approximate expressions. We eliminate ∇x_2 and $\mathbf{d}_2$ and take into account relations (2.1)–(2.5), (2.10), (2.16), (2.17), (2.22) and (2.23).

For the contribution to the stress we obtain

$$\pi^{(2)}_{\alpha\beta} = \xi_1 e_{\alpha\beta}\nabla\mathbf{u} - \xi_2\left\langle \frac{\partial}{\partial r_\alpha}\left(\frac{1}{\rho}\frac{\partial p}{\partial r_\beta}\right) + 2\frac{\partial u_\gamma}{\partial r_\alpha}e_{\gamma\beta} + \frac{\partial u_\gamma}{\partial r_\beta}\frac{\partial u_\alpha}{\partial r_\gamma}\right\rangle + \xi_3\left\langle e_{\alpha\gamma}e_{\gamma\beta}\right\rangle +$$

$$+\xi_4\left\langle\frac{\partial^2 T}{\partial r_\alpha \partial r_\beta}\right\rangle + \xi_5\left\langle\frac{\partial T}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right\rangle + \xi_6\left\langle\frac{\partial p}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right\rangle + \xi_7\left\langle\frac{\partial d_{1\beta}}{\partial r_\alpha}\right\rangle +$$

$$+\left\langle\frac{\partial T}{\partial r_\alpha}\left(\xi_8\frac{\partial x_1}{\partial r_\beta} + \xi_9 d_{1\beta}\right)\right\rangle + \left\langle d_{1\alpha}\left(\xi_{10}\frac{\partial p}{\partial r_\beta} + \xi_{11}\frac{\partial x_1}{\partial r_\beta} + \delta\xi^*_{12}d_{1\beta}\right)\right\rangle \qquad (3.1)$$

Here and below we have only taken into account those of the coefficients with an asterisk [1], that are moments of $L_i(f^{(1)}f^{(1)})$, which are non-zero in the case of Maxwell molecules. An approximate expression for $\delta\xi^*_{12}$ is given by formula (4.4) in [1]. the remaining coefficients result from the convective part of Eq. (1.1)

$$\xi_1 = -\frac{2}{3}\left\{T\frac{\partial B_i'}{\partial T} + c_i^2\frac{\partial B_i'}{\partial c_i^2}\right\}_\eta \approx \sum\frac{4}{3}\frac{\eta_i^2}{p_i}\left(\frac{7}{2} - \partial_T\eta_i\right)$$

$$\xi_2 = \{B_i'\}_\eta \approx \sum\frac{2\eta_i^2}{p_i}, \quad \xi_3 = -\frac{8}{7}\left\{c_i^2\frac{\partial B_i'}{\partial c_i^2}\right\}_\eta \approx 4\xi_2$$

$$\xi_4 = \{A_i'\}_\eta \approx \sum\frac{4}{5}\frac{\eta_i}{p_i}\psi_i, \quad \psi_i = \lambda_i' + \frac{5}{2}\omega_i\mathcal{D}_T \qquad (3.2)$$

$$\xi_5 = \left\{\frac{\partial A_i'}{\partial T}\right\}_\eta \approx \sum\frac{4}{5}\frac{\eta_i}{p_i}\frac{\partial\psi_i}{\partial T}, \quad \xi_6 = \frac{2}{\rho}\left\{\frac{\partial A_i'}{\partial c_i^2}\right\}_\eta \approx -\sum\frac{2\rho_i\eta_i}{\rho p_i^2}\omega_i\mathcal{D}_T$$

$$\xi_7 = \{\mathcal{D}_i'\}_\eta \approx \sum 2\eta_i\chi_i, \quad \chi_i = \frac{\omega_i}{kn_i}\mathcal{D}_{12} - \frac{\delta\gamma_i}{n}$$

$$\xi_8 = \left\{\frac{\partial A_i'}{\partial x_1}\right\}_\eta \approx \sum\frac{8\eta_i}{p_i}\frac{\partial\psi_i}{\partial x_1}, \quad \xi_9 = \left\{\frac{\partial\mathcal{D}_i'}{\partial T}\right\}_\eta \approx \sum\frac{2\eta_i}{T}\frac{\partial(T\chi_i)}{\partial T}$$

$$\xi_{10} = \frac{2}{\rho}\left\{\frac{\partial\mathcal{D}_i'}{\partial c_i^2}\right\}_\eta \approx -\sum\frac{2m_i\eta_i}{\rho k p_i}\omega_i\mathcal{D}_{12}, \quad \xi_{11} = 2\left\{\frac{\partial\mathcal{D}_i'}{\partial x_1}\right\}_\eta \approx \sum\frac{4\eta_i}{n_i}\frac{\partial(n_i\chi_i)}{\partial x_1}$$

In relations (3.2) and below the first expressions for the coefficients ξ, γ, δ are accurate while the second expressions are approximate; the latter are obtained within the framework of approximation (2.11) and (2.16). The summation is carried out from $i = 1$ to $i = 2$, and we have introduced the notation $\partial_T N \equiv \partial \ln N/\partial \ln T$.

In the operators $\{\ \}_\eta$, $\{\ \}\gamma$ determined previously [1]), Ω must be replaced by i.

The expressions for the contributions to the vector transport properties can be combined as follows. We will denote these contributions by

$$\Lambda_\alpha^{(2)} = \varphi_1 \frac{\partial T}{\partial r_\alpha}\nabla\mathbf{u} + \varphi_2\left[\frac{1}{3}\frac{\partial(T\nabla\mathbf{u})}{\partial r_\alpha} + \frac{\partial u_\beta}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right] + \left(\varphi_3\frac{\partial p}{\partial r_\beta} + \varphi_4\frac{\partial T}{\partial r_\beta}\right)e_{\beta\alpha} + \varphi_5\frac{\partial e_{\alpha\beta}}{\partial r_\beta} +$$

$$+\varphi_6 d_{1\alpha}\nabla\mathbf{u} - \varphi_7\left[2\frac{\partial u_\beta}{\partial r_\alpha}d_{1\beta} + \frac{5}{3}x_1x_2(m_2 - m_1)\frac{n}{\rho}\frac{\partial\nabla\mathbf{u}}{\partial r_\alpha}\right] + \left[\varphi_8\frac{\partial x_1}{\partial r_\beta} + (\varphi_9 + \delta\varphi_9^*)d_{1\beta}\right]e_{\alpha\beta} \tag{3.3}$$

The coefficients φ_m ($m = 1, 2, \dots, 9$) are given by the expressions

$$\varphi_1 = -\frac{2}{3}\left\{T\frac{\partial A_i'}{\partial T} + c_i^2\frac{\partial A_i'}{c_i^2}\right\}_\varphi \approx \sum\frac{14m_i}{15kp_i}\left[b_i^{(1)}\lambda_i'\left(1 - \frac{2}{7}\partial_T\lambda_i'\right) - b_i^{(0)}\omega_i\mathcal{D}_T(1 - \partial_T\mathcal{D}_T)\right]$$

$$\varphi_2 = -2\{A_i'\}_\varphi \approx -\sum\frac{4m_i}{5kp_i}(b_i^{(1)}\lambda_i' - b_i^{(0)}\omega_i\mathcal{D}_T) \tag{3.4}$$

$$\varphi_3 = \frac{2}{\rho}\left\{B_i' + \frac{2}{5}c_i^2\frac{\partial B_i'}{\partial c_i^2}\right\}_\varphi \approx -\sum\frac{4m_i\eta_i}{5\rho kp_i}b_i^{(1)}$$

$$\varphi_4 = \frac{2}{5}\left\{c_i^2\left(\frac{\partial B_i'}{\partial T} + \frac{\partial A_i'}{\partial c_i^2}\right)\right\}_\varphi \approx$$

$$\approx \sum\frac{4\eta_i}{5p_i}\left\{b_i^{(1)}\partial_T(T^{7/2}\eta_i) - b_i^{(0)}\partial_T\eta_i + \frac{m_i}{k\eta_i}\left[\frac{7}{5}b_i^{(1)}\lambda_i' - (b_i^{(0)} - b_i^{(1)})\omega_i\mathcal{D}_T\right]\right\}$$

$$\varphi_5 = \frac{2}{5}\{c_i^2B_i'\}_\varphi \approx \sum\frac{4\eta_i}{5kn_i}(b_i^{(1)} - b_i^{(0)})$$

$$\varphi_6 = -\frac{2}{3}\left\{T\frac{\partial\mathcal{D}_i'}{\partial T} + c_i^2\frac{\partial\mathcal{D}_i'}{\partial c_i^2}\right\}_\varphi \approx$$

$$\approx \sum\frac{2m_i}{3k}\left\{b_i^{(0)}\frac{\omega_i}{kn_i}\mathcal{D}_{12}\left(-1 + \frac{2}{5}\partial_T\mathcal{D}_{12}\right) + b_i^{(1)}\frac{\delta\gamma_i}{n}\left(-\frac{5}{2} + \partial_T\delta\gamma_i\right)\right\}$$

$$\varphi_7 = \{\mathcal{D}_i'\}_\varphi \approx -\sum\frac{m_i}{k}\left(\frac{2}{5}b_i^{(0)}\frac{\omega_i}{kn_i}\mathcal{D}_{12} + b_i^{(1)}\frac{1}{n}\delta\gamma_i\right)$$

$$\varphi_8 = \frac{4}{5}\left\{c_i^2\frac{\partial B_i'}{\partial x_1}\right\}_\varphi \approx \sum\frac{8}{5kn_i}\frac{\partial\eta_i}{\partial x_1}(b_i^{(1)} - b_i^{(0)})$$

$$\varphi_9 = -\frac{4}{5}\left\{c_i^2\frac{\partial\mathcal{D}_i'}{\partial c_i^2}\right\}_\varphi \approx \sum\frac{4m_i}{5k}\left(\frac{\omega_i}{kn_i}\mathcal{D}_{12}(b_i^{(1)} - b_i^{(0)}) - \frac{7}{2n}b_i^{(1)}\delta\gamma_i\right)$$

The factor ω_i is defined by the last expression of (2.10).

In order to obtain relations for the contribution to the component of reduced thermal flux $h_\alpha^{(2)}$, we must make the replacement

$$\Lambda_\alpha^{(2)} \to h_\alpha^{(2)}, \quad \{\ \}_\varphi \to \{\ \}_\gamma, \quad \varphi_m \to \gamma_m, \tag{3.5}$$

(the quantity $\delta\gamma_9^*$ is given by formula (4.4) from [1]) and put

$$b_i^{(0)} = -\frac{5}{2}\omega_i\mathcal{D}_T, \quad b_i^{(1)} = \lambda_i' \tag{3.6}$$

To determine the contribution to the component of the diffusion velocity $V_{1\alpha}^{(2)}$, as in (3.5) we must make the following replacements

$$\Lambda_\alpha^{(2)} \to V_{1\alpha}^{(2)}, \quad \{\ \}_\varphi \to \{\ \}_\delta, \quad \varphi_m \to \delta_m, \quad \delta\varphi_9^* \to 0 \tag{3.7}$$

When calculating the transport coefficients in the expression for $V_{1\alpha}^{(2)}$ it is more convenient, using formula (1.24) from [1], to obtain the corresponding formula for the difference in the diffusion velocities, and then use the relation between them. As a result we obtain

$$V_{1\alpha}^{(2)} = \frac{\rho_2}{\rho}(V_{1\alpha}^{(2)} - V_{2\alpha}^{(2)}) = -\frac{\rho_2}{\rho n^2}\sum\int \mathfrak{D}_i c_{i\alpha} M_i d\mathbf{c}_i \tag{3.8}$$

Hence, the operator has the form

$$\{N\}_\delta = -\frac{2kT}{3n^2}\sum\int \frac{N\rho_2}{m_i\rho}\mathfrak{D}_i w_i^2 d\mathbf{c}_i \tag{3.9}$$

instead of (2.16) from [1] (in which there is an error: the extraneous factor N is included under the integral).

Finally, instead of (3.6), when calculating $V_{1\alpha}^{(2)}$ taking (3.8) and (3.9) into account, we must put

$$b_i^{(0)} = -\frac{5\rho_2\omega_i}{2\rho p}\mathfrak{D}_{12}, \quad b_i^{(1)} = -\frac{5\rho_2 x_i}{2nT\rho}\delta\gamma_i \tag{3.10}$$

Hence, the approximate values of the Burnett coefficients in the components of the diffusion velocity $V_{1\alpha}^{(2)}$ can be calculated using formulae (3.4), (3.7) and (3.10).

We emphasize, that despite the differences, we have retained here the notation for the transport coefficients ξ_m, γ_m and δ_m [1]. In expression (4.5) of [1] for α_3 we must put $\rho_2/\rho_1 + 1$ instead of $\rho_2/\rho_1 - 1$.

The formulae in [1] for a binary mixture follow from (3.1)–(3.10) if we put $\mathfrak{D}_T = 0$, $\delta\gamma_i = 0$, $\mathbf{d}_i = \nabla x$ in them.

4. THE CONTRIBUTIONS OF BURNETT'S APPROXIMATION TO THE TRANSPORT PROPERTIES OF A POLYATOMIC GASES

The contributions to the scalar part of the stress tensor, divergence-free stresses and the heat flux here take the form

$$\Pi^{(2)} = (\omega_1 + \omega_1^*)e_{\alpha\beta}e_{\beta\alpha} + \omega_2\nabla^2 T + (\omega_3 + \omega_3^*)(\nabla T)^2 + (\omega_4 + \omega_4^*)(\nabla\mathbf{u})^2 -$$

$$-\omega_5\left[\frac{\partial}{\partial r_\alpha}\left(\frac{1}{\rho}\frac{\partial p}{\partial r_\alpha}\right) + \frac{\partial u_\beta}{\partial r_\alpha}\frac{\partial u_\alpha}{\partial r_\beta}\right] + \omega_6\frac{1}{\rho}\frac{\partial p}{\partial r_\alpha}\frac{\partial T}{\partial r_\alpha} \tag{4.1}$$

$$\pi_{\alpha\beta}^{(2)} = (\xi_1 + \xi_1^*)e_{\alpha\beta}\nabla\mathbf{u} - \xi_2\left\langle\frac{\partial}{\partial r_\alpha}\left(\frac{1}{\rho}\frac{\partial p}{\partial r_\beta}\right) + 2\frac{\partial u_\gamma}{\partial r_\alpha}e_{\gamma\beta} + \frac{\partial u_\gamma}{\partial r_\beta}\frac{\partial u_\alpha}{\partial r_\gamma}\right\rangle + (\xi_3 + \xi_3^*)e_{\alpha\gamma}e_{\gamma\beta} +$$

$$+\xi_4\left\langle\frac{\partial^2 T}{\partial r_\alpha\partial r_\beta}\right\rangle + (\xi_5 + \xi_5^*)\left\langle\frac{\partial T}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right\rangle + \frac{\xi_6}{\rho}\left\langle\frac{\partial p}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right\rangle \tag{4.2}$$

$$q_\alpha^{(2)} = (\gamma_1 + \gamma_1^*)\frac{\partial T}{\partial r_\alpha}\nabla\mathbf{u} + 2\gamma_2\left[\frac{\partial}{\partial r_\alpha}\left(\frac{T}{3c_v^*}\nabla\mathbf{u}\right) + \frac{\partial u_\beta}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right] + \frac{\gamma_3}{\rho}\frac{\partial p}{\partial r_\beta}e_{\beta\alpha} +$$

$$+(\gamma_4 + \gamma_4^*)\frac{\partial T}{\partial r_\beta}e_{\beta\alpha} + \gamma_5\frac{\partial e_{\alpha\beta}}{\partial r_\beta} + \gamma_{10}\frac{1}{\rho}\frac{\partial p}{\partial r_\alpha}\nabla\mathbf{u} + \gamma_{12}\frac{\partial\nabla\mathbf{u}}{\partial r_\alpha} \tag{4.3}$$

The coefficients with an asterisk occurring in expressions (4.1)–(4.3) in the case of a monatomic gas are small and will be neglected [2]; as confirmed in [7], they are also negligibly small for classical models of molecules with rotational degrees of freedom (rough and loaded spheres). In general, this conclusion, of course, needs to be checked. The general approximate expression [1] for the coefficients without asterisks reduce to the form

$$\omega_1 = \frac{2}{p}\eta\zeta, \quad \omega_2 = \frac{\zeta}{p}(\lambda_t - \sigma\lambda_v)$$

$$\omega_3 = \frac{\zeta}{p}\left[\frac{\partial\lambda_t}{\partial T} - \sigma c_v T^2 \frac{\partial}{\partial T}\left(\frac{\lambda_v}{c_v}T^{-2}\right) - \lambda_v \frac{\sigma k}{c_v T}\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]$$

$$\omega_4 = \frac{\zeta^2}{p}\left\{\frac{5}{2} + \sigma^2\frac{c_v}{k} - \frac{2}{3c_v^*}\left[\frac{3}{2}\partial_T\zeta + \sigma^2\frac{c_v}{k}(\partial_T(\zeta\sigma) - 2) + \sigma^2\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]\right\}$$

$$\omega_5 = \frac{\zeta^2}{p}\left(\frac{3}{2} + \sigma^2\frac{c_v}{k}\right), \quad \omega_6 = 0, \quad \xi_1 = \frac{4}{3}\frac{\eta^2}{p}\left(\frac{7}{2} - \frac{1}{c_v^*}\partial_T\eta + \frac{3}{2}\frac{\zeta}{\eta}\right)$$

$$\xi_2 = \frac{2\eta^2}{p}, \quad \xi_3 = 4\xi_2, \quad \xi_4 = \frac{4}{5}\frac{\eta}{p}\lambda_t, \quad \xi_5 = \frac{4}{5}\frac{\eta}{p}\frac{\partial\lambda_t}{\partial T}, \quad \xi_6 = 0 \tag{4.4}$$

$$\gamma_1 = \frac{4m}{15pkc_v^*}\left\{-\lambda_t^2\partial_T\lambda_t - \frac{5}{2}\lambda_v\left[T^3\frac{\partial}{\partial T}\left(\frac{k\lambda_v}{T^2 c_v}\right) + \lambda_v\frac{k^2}{c_v^2}\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]\right\} +$$

$$+ \frac{\zeta}{p}\left\{\lambda_t\partial_T(\zeta T^2) + \lambda_v[1 - \sigma\partial_T(\zeta\sigma T^{-1})] - \lambda_v\sigma\frac{k}{c_v}\langle(\Delta\varepsilon_\omega)^3\rangle_c\right\} + \frac{2m}{3kp}\left(\frac{7}{5}\lambda_t^2 + \frac{5}{2}\frac{k}{c_v}\lambda_v^2\right)$$

$$\gamma_2 = -\frac{2m}{5kp}\left(\lambda_t^2 + \frac{5}{2}\frac{k}{c_v}\lambda_v^2\right), \quad \gamma_3 = -\frac{4m\eta}{5kp}\lambda_t$$

$$\gamma_4 = \frac{4\eta}{5p}\left\{\lambda_t\partial_T(T^{7/2}\eta) + \frac{5}{2}\lambda_v + \frac{m}{k\eta}\left(\frac{7}{5}\lambda_t^2 + \frac{5}{2}\frac{k}{c_v}\lambda_v^2\right)\right\}$$

$$\gamma_5 = \frac{4\eta\lambda_t}{5kn}, \quad \gamma_{10} = -\frac{m}{kT}\gamma_{12}, \quad \gamma_{12} = \frac{\zeta}{nk}(\lambda_t - \sigma\lambda_v)$$

$$\sigma = \frac{3k}{2c_v}, \quad \zeta = \frac{1}{4}\pi k c_v \eta Z\left(\frac{3}{2}k + c_v\right)^{-2}$$

Hence c_v is the heat capacity due to the internal degrees of freedom of the molecules at constant volume, the operation $\langle\ldots\rangle_c$ was defined previously in [1], expressions for σ and ζ are given corresponding to known results [5, 8], and Z is the characteristic ratio of the relaxation times of the internal and translational degrees of freedom of the molecules. The dynamic coefficient of viscosity η in many cases differs only slightly from the case of a monatomic gas, and hence the translational thermal conductivity λ_t, the internal thermal conductivity λ_v and the overall thermal conductivity λ can be conveniently written as

$$(\lambda_t, \lambda_v, \lambda) = \frac{15}{4}R\eta(\Lambda_t, \Lambda_v, \Lambda_\Sigma), \quad \Lambda_\Sigma = \Lambda_t + \Lambda_v, \quad R = \frac{k}{m} \tag{4.5}$$

For a monatonic gas $\Lambda_t = \Lambda_\Sigma = 1$, $\Lambda_v = 0$

In the widely used Mason–Monchik approximation [3, 5, 8], the coefficient η is the same as for the corresponding monatomic gas, while for the thermal conductivities we have [5]

$$\Lambda_t = 1 - \frac{A}{\sigma}, \quad \Lambda_v = \frac{2\beta}{5\sigma}(1 + A)$$

$$A = \frac{5 - 2\beta}{\pi Z}\left[1 + \frac{2}{\pi Z}\left(\frac{5}{2\sigma} + \beta\right)\right]^{-1}, \quad \beta = \frac{\rho \mathcal{D}}{\eta}\varphi(Z) = 1.328\varphi(Z) \tag{4.6}$$

Here $\mathcal{D}$ is the self-diffusion coefficient of a monatomic gas.

We will consider in more detail such diatomic gases as nitrogen or oxygen for those values of T when only rotational degrees of freedom of the molecules are excited. Summation over the rotational quantum numbers $\omega \equiv J$ in the formulae for the mean values $\langle \ldots \rangle_c$ can be replaced here by integration over J (the quasiclassical approximation, $J \gg 1$). Following, for example, the well-known approach in [8], we obtain

$$c_v = k, \quad \sigma = \frac{3}{2}, \quad \langle (\Delta\varepsilon_\omega)^3 \rangle_c = 2$$

The characteristic ratio of the rotational and translational relaxation times is given by the approximate Parker formula [8]

$$Z = Z^\infty\left[1 + \frac{\pi^{3/2}}{2}\theta^{1/2} + \left(\frac{\pi^2}{4} + 2\right)\theta\right]^{-1}, \quad \theta = \frac{T_*}{T} \tag{4.7}$$

where $T_* = 91.5$ K for nitrogen and $T_* = 88$ K for oxygen. For nitrogen the values $Z^\infty \approx 18$–22 correspond to the range of experimental data. The ratio $\varphi(Z)$ of the coefficients of self-diffusion of a diatomic and a monatomic gas for small Z, occurring in the last formula of (4.6), differs considerably from unity. This difference can be estimated using Sandler's formula [8]

$$\varphi(Z) = 1 + 0.27Z^{-1} - 0.44Z^{-2} - 0.90Z^{-3} \tag{4.8}$$

5. THE EQUATIONS OF THERMAL STRESS CONVECTION OF POLYATOMIC GAS

Following the approach described earlier [1, Section 5], we will write the variable part of the total stress tensor, which will occur only in the momentum equation and in the expression for the surface force, in the form

$$p_0 \delta p\, \delta_{\alpha\beta} + \tau_{\alpha\beta} \equiv X\delta_{\alpha\beta} + R\pi_{\alpha\beta} \tag{5.1}$$

$$X = p_0\delta p - \zeta\nabla\mathbf{u} + \omega_2\nabla^2 T + (\omega_3 + \omega_3^*)(\nabla T)^2 + \frac{2}{3}\eta\nabla\mathbf{u} - \frac{1}{3}\xi_4\nabla^2 T - \frac{1}{3}(\xi_5 + \xi_5^*)(\nabla T)^2$$

$$R\pi_{\alpha\beta} = -\eta\left(\frac{\partial u_\alpha}{\partial r_\beta} + \frac{\partial u_\beta}{\partial r_\alpha}\right) + \xi_4\frac{\partial^2 T}{\partial r_\alpha \partial r_\beta} + (\xi_5 + \xi_5^*)\frac{\partial T}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}$$

We recall that in the case considered

$$u \sim V_0, \quad V_0 = \frac{\eta_0}{\rho_0 L}, \quad p = p_0(1 + \delta p), \quad p_0 = \text{const}, \quad \delta p = O(\text{Kn}^2), \quad \text{Kn} = \frac{V_0}{\sqrt{RT_0}}$$

the gradients of T are of the order of unity, the zero subscript denotes characteristic values, δp is the relative variable part of the pressure, L is a characteristic dimension, and Knudsen's number $\text{Kn} \ll 1$.† Terms of the Chapman–Enskog expansion of the order of Kn^2 compared with those written down are ignored in relations (5.1).

†The structure of the transient equations of thermal stress convection of a polyatomic gas have been considered in the following paper: GALKIN, V. S., KOGAN, M. N. and FRIDLENDER, O. G., Thermal- and diffusion-stress phenomena. In *Proceedings of the Fourth All-Union Conference on the Dynamics of Rarefied Gases and Molecular Gas Dynamics.* Izd. Otdel TsAGI, Moscow, 1977, 321–322; however, the non-physical assumption was made that p_0 is a specified function of t, which complicates the equations.

The specific feature of the problem is such that X is a new gas-dynamic variable, the data on the structure of which are not needed. In the expressions for ω and ξ we have put $p = p_0$, so that they depend on T and are independent of p. Hence

$$\frac{\partial}{\partial r_\beta}\left(\xi_4 \frac{\partial^2 T}{\partial r_\alpha \partial r_\beta}\right) = \frac{\partial}{\partial r_\alpha}\left[\xi_4 \nabla^2 T + \frac{1}{2}\frac{d\xi_4}{dT}(\nabla T)^2\right] - \frac{1}{2}\frac{d^2\xi_4}{dT^2}(\nabla T)^2 \frac{\partial T}{\partial r_\alpha} - \frac{d\xi_4}{dT}\nabla^2 T \frac{\partial T}{\partial r_\alpha}$$

$$\frac{\partial}{\partial r_\beta}\left(\xi \frac{\partial T}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta}\right) = \frac{\partial}{\partial r_\alpha}\left[\frac{\xi}{2}(\nabla T)^2\right] + \frac{1}{2}\frac{d\xi}{dT}(\nabla T)^2 \frac{\partial T}{\partial r_\alpha} + \xi \nabla^2 T \frac{\partial T}{\partial r_\alpha}, \quad \xi = \xi_5 + \xi_5^* \qquad (5.2)$$

For the final formulation of the required equations we refer the quantities u, T, ρ, η, X, r, t to V_0, T_0, ρ_0, η_0, V_0^2, L, L/V_0 respectively, retaining the previous notation as the dimensionless variables. We will neglect terms $O(\mathrm{Kn}^2)$ compared with the unity and take expressions (4.5) into account. The equations of state, continuity and energy take the form

$$\rho T = 1, \quad \frac{D \ln T}{Dt} = \nabla \mathbf{u} \qquad (5.3)$$

$$E\nabla \mathbf{u} = \nabla(\eta\Lambda_\Sigma \nabla T) \equiv \frac{d\eta\Lambda_\Sigma}{dT}(\nabla T)^2 + \eta\Lambda_\Sigma \nabla^2 T, \quad E = \frac{4}{15}\left(\frac{5}{2} + \frac{c_v}{k}\right) \qquad (5.4)$$

The derivative with respect to t in energy equation (5.4) is eliminated by using the second equation of (5.3).

The temperature stresses can be written in dimensionless variables in the form

$$\alpha_1 \left\langle \frac{\partial^2 T}{\partial r_\alpha \partial r_\beta} \right\rangle + \alpha_2 \left\langle \frac{\partial T}{\partial r_\alpha}\frac{\partial T}{\partial r_\beta} \right\rangle \qquad (5.5)$$

Taking relations (5.1)–(5.5) into account, the divergent terms in (5.2) $\partial/\partial r_\alpha[\ldots]$ are combined with X while $\nabla^2 T$ is eliminated using relations (5.4)), we transform the momentum equation

$$\frac{1}{T}\left(\frac{Du_\alpha}{Dt} - F_\alpha\right) + \frac{\partial W}{\partial r_\alpha} = \frac{\partial}{\partial r_\beta}\eta\left(\frac{\partial u_\alpha}{\partial r_\beta} + \frac{\partial u_\beta}{\partial r_\alpha}\right) + Y_T(\nabla T)^2 \frac{\partial T}{\partial r_\alpha} + \frac{E}{\eta\Lambda_\Sigma} U_T \frac{\partial T}{\partial r_\alpha}\nabla \mathbf{u} \qquad (5.6)$$

$$W = X + \alpha_1 \frac{E\nabla \mathbf{u}}{\eta\Lambda_\Sigma} + X_T(\nabla T)^2$$

$$X_T = \frac{1}{2}\left(\alpha_2 + \frac{d\alpha_1}{dT} - \frac{2\alpha_1}{\eta\Lambda_\Sigma}\frac{d\eta\Lambda_\Sigma}{dT}\right) \qquad (5.7)$$

$$Y_T = \frac{1}{2}\left(\frac{d^2\alpha_1}{dT^2} - \frac{d\alpha_2}{dT} + \frac{2}{\eta\Lambda_\Sigma}\left(\alpha_2 - \frac{d\alpha_1}{dT}\right)\frac{d\eta\Lambda_\Sigma}{dT}\right), \quad U_T = \frac{d\alpha_1}{dT} - \alpha_2$$

An equation for the momentum for monatomic gas of a similar form was previously obtained in [9] (incidentally, we have corrected errors here), where $\mathbf{F}$ is the dimensionless external force [1]. We can formally assume p_0 to be a known function of t [10], in which case the equations of the thermal stress convection have a more complex form.

In Eq. (5.6) W is a new dependent variable. The inclusion of certain terms of the divergence of the temperature stresses in the expression for W leads to the fact that the order of the equation of the momentum does not change when these stresses are taken into account.

We will analyse the effect of the rotational degrees of freedom of diatomic molecules on the coefficients of Eq. (5.6), expressed in terms of Burnett's transport coefficients. As in Section 4 we will omit ξ_5^* (see (4.2)). Using expressions (4.4) for ξ_4 and ξ_5 and formula (4.5), we obtain

$$\alpha_1 = 3\eta^2\Lambda_t, \quad \alpha_2 = 3\eta\frac{d\eta\Lambda_t}{dT} \qquad (5.8)$$

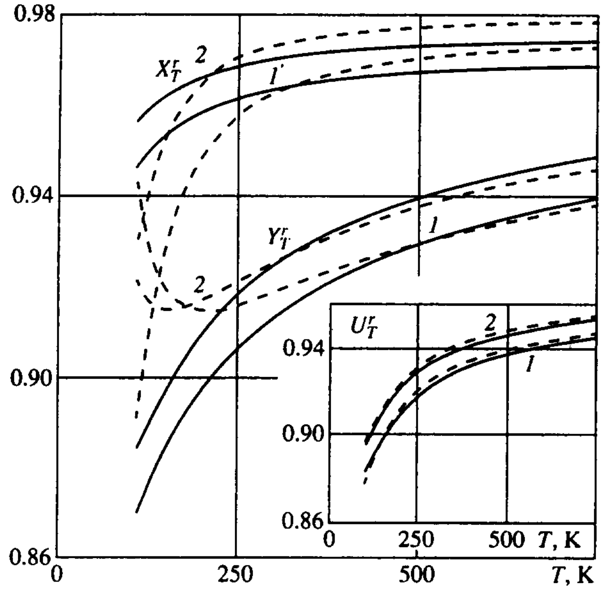

Fig. 1

We will use approximation (4.6) assuming $\sigma = 3/2$ and using formulae (4.7) and (4.8) with $T_* = 9.15$ K. We will assume that the dynamic coefficient of viscosity is independent of the rotational degrees of freedom of the molecules and is power function of T, i.e. $\eta = T^s$. Then, taking relations (5.7) and (5.8) into account, we obtain

$$
\begin{gathered}
X_T = \frac{3s}{2} T^{2s-1} X_T^r, \quad X_T^r = \Lambda_t \left(1 + \frac{2T}{s} \frac{d}{dT} \ln \frac{\Lambda_t}{\Lambda_\Sigma} \right) \\
Y_T = -\frac{3s}{2} T^{2s-2} Y_T^r, \quad Y_T^r = \Lambda_t \left(1 + T \frac{d}{dT} \ln \frac{\Lambda_\Sigma^2}{\Lambda_t} \right) \qquad (5.9) \\
U_T = 3sT^{2s-1} U_T^r, \quad U_T^r = \Lambda_t
\end{gathered}
$$

In the case of a monatomic gas $X_T^r = Y_T^r = U_T^r = 1$. The coefficients X_T and Y_T are the most important in the theory of thermal stress convection [9], but they depend in a most complex way on the polyatomicity. The results of calculations are shown in the figure, the continuous curves corresponding to $\varphi = 1$ and the dashed curves corresponding to the relations $\varphi(Z)$ from formula (4.8), where the numbers 1 and 2 are for values of $Z^\infty = 18$ and 22. For $X_T^r(T)$ we assumed $s = 1$, which holds for low T.

The value of φ is found to differ from unity at low temperature $T \leqslant 200$ K. If $\varphi(Z)$ has a variable value, the effect of the rotational degrees of freedom of the molecules manifests itself to the maximum extent for the $X_T^r(T)$ and $X_T^r(T)$ curves. The dependences on T obtained can be qualitatively explained by the fact that as T increases the value of Z increases, and the so-called Euken approximation becomes valid, when the temperature stress coefficients are given by the expressions for monatomic gas, i.e. X_T^r, Y_T^r and U_T^r approach unity [1, Section 3].

Hence, the effect of the rotational degrees of freedom of diatomic molecules (nitrogen, oxygen, etc.) on the coefficients of the terms of the equations for thermal stress convection, resulting from Burnett temperature stresses, is considerable at low temperatures (approximately up to 10% at $T = 100$ K).

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REFERENCES

1. GALKIN, V. S., Burnett's equations for multicomponent mixtures of polyatomic gases. *Prikl. Mat. Mekh.*, 2000, **64**, 4, 590–604.
2. CHAPMAN, S. and COWLING, T. G., *The Mathematical Theory of Non-uniform Gases*. Cambridge University Press, Cambridge, 1952.
3. FERZIGER, J. H. and KAPER, H. G., *Mathematical Theory of Transport Processes in Gases*. North Holland, Amsterdam, 1972.
4. ALEKSANDROV, V., BORIS, A., FREEDLENDER, O., KOGAN, M., NIKOLSKY, Yu. and PERMINOV, V., Thermal stress effect and its experimental detection. *Rarefied Gas Dynamics: proc. 20th Int. Symp. Beijing, China, 1996*. Peking Univ. Press, 1997, 79–84.
5. GALKIN, V. S., and ZHAROV, V. A., The solution of problems of the sound propagation and the structure of a weak shock wave in a polyatomic gas using Burnett's equations. *Prikl. Mat. Mekh.*, 2001, **65**, 3, 467–476.
6. VOLKOV, I. V., and GALKIN, V. S., Analysis of the slip coefficients and the temperature jump in a binary mixture of gases. *Izv. Akad. Nauk SSSR. MZhG*, 1990, 6, 152–159.
7. MCCOY, B. J. and DAHLER, J. S., Second-order constitutive relations for polyatomic fluids. *Phys. Fluids*, 1969, **12**, 1392–1403.
8. ZHDANOV, V. M. and ALIYEVSKII, M. YA., *Transport and Relaxation Processes in Molecular Gases*. Nauka, Moscow, 1989.
9. GALKIN, V. S. and FRIDLENDER, D. G., The forces on a body in a gas due to Burnett stresses. *Prikl. Mat. Mekh.*, 1974, **38**, 2, 271–283.
10. GALKIN, V. S., Derivation of the equations of the slow flows of gas mixtures from Boltzmann's equation. *Uch. Zap. TsAGI*, 1974, **5**, 4, 40–47.

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